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CALCULATION OF RESIDUAL STRESSES INDUCED DURING LASER QUENCH-HARDENING OF STEEL

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We present a theoretical and numerical analysis of the quasi-stationary uncoupled problem of thermoelastic—plasticity with the goal of estimating the amount of residual stress in steel after laser quench-hardening.

During laser quench-hardening of steels, temperature and concentration gradients and also structural phase transformations lead to the inception and development of temperature-, concentration- and phase-stresses, respectively, in the hardened layer of the metal. These are in turn imposed on the initial structure of the material, which in general is deformed. As a result, during heating the conditions for microscopic plastic deformation are created [1]. By changing the activation energy of the processes, this deformation significantly influences the kinetics of laser austenizing, carbide decay, and the diffusion of carbon and alloying elements out of the carbides in the matrix, and finally leads to a shift of the instrumental start of the (α–γ) phase change. Residual stresses are formed in the surface layer during rapid cooling. The mechanical properties of the laser quench-hardened layer depend to a significant degree on these residual stresses.

At present, analysis of the thermoelastic behavior of a solid body during laser heating has been widely developed [2-4], based on the simultaneous solution of the uncoupled problems of thermal conductivity and thermoelasticity. Fewer works are devoted to the theoretical investigation of phase stresses and plastic deformation during laser heating [1]. However, due to the difficulty of high-temperature γ-phase diagnostics, comparison with experiment for laser heating is problematic. On the other hand, there are well-known works on laser cooling [5-7], where the residual stresses in the hardened layer have been determined using x-radiography. The stresses were determined in this layer as a functon of depth in the zone of laser influence (ZLI) and as a function of the spot of laser action (LA) on the surface. There are only a few attempts to theoretically analyze the residual stresses during laser cooling [8, 9]. For example, in [8] only the martensite phase (γ–α) residual stresses after LA are analyzed, and in [3] thermal stresses calculated for the heating stage in a stationary approximation are called residual stresses. Moreover in both cases [3, 8], residual stress calculation was done within the framework of elasticity theory, which is physically unfounded. The most complete approach is in [9], where a method developed earlier for computing residual thermal stresses in welded joints is carried over to laser heat-treating of steel. This method takes the theory of plastic deformation into account. Thus the development of a theoretical and numerical method of computing stresses is a topical problem in laser production technology. Its solution will permit more exact prediction of the mechanical characteristics of the layer modified by laser radiation (LR).

The model of laser quench-hardening developed by us earlier [1, 10] creates a physically clear picture of the thermal, kinetic, and diffusion processes in time and space during heating. We examine the interconnection of these processes and their influence on the development and redistribution of stresses during LA. Let an axisymmetric beam flux with Gaussian intensity distribution in the laser beam cross section fall on the surface of the steel.

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Let the energy absorption take place immediately at the surface (which is true for $I_0 \leqslant 10^6$ W/cm^2). We will subscript all variables in the heating stage with j = 1, and those of the cooling stage with j = 2. Here $T_j < T_{mp}$ is always true. Then the general form of the equation of thermal conductivity, including initial and boundary conditions, for an uncoupled thermoelastic problem in cylindrical coordinates is

$$\partial T_j/\partial t = a\,(\partial^2 T_j/\partial r^2 + 1/r\cdot\partial T_j/\partial r + \partial^2 T_j/\partial z^2), \tag{1}$$

$$-\lambda_s \partial T_j/\partial z|_{z=0} = \begin{cases} AI_m = AI_0 \exp(-r^2/r_n^2), & j=1,\ 0 \leqslant t \leqslant \tau_d, \\ 0, & j=2,\ t > \tau_d, \end{cases} \tag{2}$$

$$T_j(r,\ z \to \infty;\ t) = T_{1,0}(r,\ z;\ t=0) = 0,\ T_2(r,\ z;\ t=\tau_d) = T_1(r,\ z;\ t=\tau_d). \tag{3}$$

In (1)-(3) we assume to a first approximation that the thermophysical constants are independent of temperature and are the same for the α- and γ-phases. Since for $I_0 \leqslant 10^6$ W/cm^2 the ratio of the rates of change of strain and temperature is small [11], then for the LA cases of interest it is valid to discard the bulk term on the right side of (1), which describes the change in temperature due to mechanical deformation. This uncouples the problem.

The solution to (1)-(3) is found by the Laplace and Hankel integral transforms. Thus during the heating stage the solution is of the form:

$$T_{1;\delta,s}(\delta,\ z,\ s) = \frac{AI_0 r_n^2 \exp(-\delta^2 r^2/4 - z\sqrt{\delta^2 + s/a})}{2\lambda_s \sqrt{\delta^2 + s/a}}, \tag{4}$$

where δ, s are the integration variables of the Hankel and Laplace transforms, respectively. By applying the inverse transform, we obtain

$$T_1(r,\ z,\ t) = AI_0 r_n^2/4\lambda_s \int_0^\infty \exp(-\delta^2 r_n^2/4)\, j_0(\delta r)\, dr\, * \\ * [\exp(-z\delta)\operatorname{erfc}((z-2\delta a t)/2\sqrt{at}) - \exp(z\delta)\operatorname{erfc}((z+2\delta a t)/2\sqrt{at})]. \tag{5}$$

We note that solution (4) is analogous to that found in [2]. Using (5) as an initial condition, the cooling stage of the solution takes on the form

$$T_{2;\delta,s}(\delta,\ z,\ s) = \frac{ch(z\sqrt{\delta^2 + s/a})}{a\sqrt{\delta^2 + s/a}} \int_0^\infty T_{1,\delta}(\xi,\ \delta,\ \tau_d) \exp(-\xi\sqrt{\delta^2 + s/a})\, d\xi \tag{6}$$

or after applying the inverse transform:

$$T_2(r,\ z,\ t) = \frac{AI_0 r_n^2}{8\lambda_s \sqrt{a\pi(t-\tau_d)}} \int_0^\infty \exp(-\delta^2 (a(t-\tau_d) + r_n^2/4) j_0(\delta r)\, d\delta) \times \\ \times \int_0^\infty d\xi\, [\exp(-(z-\xi)^2/4a(t-\tau_d)) + \exp(-(z+\xi)^2/4a(t-\tau_d))]\, [\exp(-\xi\delta) \times \\ \times \operatorname{erfc}((\xi - 2\delta a\tau_d)/2\sqrt{a\tau_d}) - \exp(\xi\delta)\operatorname{erfc}((\xi + 2\delta a\tau_d)/2\sqrt{a\tau_d})], \tag{7}$$

The diffusion redistribution of carbon in the ZLI, that is, in the volume of metal from the irradiated surface to a boundary whose position is determined by the isotherm $T = T_{sa}$, was also numerically analyzed by us in [1, 10]. The fundamental difference between the diffusion problem posed in [1, 10] and that of earlier known works [12, 13] is the transfer of the carbon source in the austenite due to decaying carbides from the boundary conditions to the volume term in the diffusion equation. Such an approach allows us to find the carbon concentration C as a continuous function of ZLI coordinates, which then makes it possible to use it in subsequent calculation of the mechanical properties of steel (e.g., the hardness or the residual stresses). The traditional approach [12, 13] in general very clearly solves the diffusion problem for an individual austenite grain, but without indicating explicitly the location of this grain in the ZLI. As a result, such a solution must be matched at the grain boundaries, which is a quite complicated task.

So the diffusion equations for carbon in the γ-phase with initial and boundary conditions is written:

$$\partial C/\partial t = D(\partial^2 C/\partial r^2 + 1/r\cdot\partial C/\partial r + \partial^2 C/\partial z^2) + \psi\partial C_i^k(r, z, t)/\partial t, \tag{8}$$

$$0 \leqslant r, z \leqslant R_0,\ Z_0(\text{ZLI});\ t_{sa} \leqslant t \leqslant \tau_d,$$

$$C(r, z, t = t_{sa}) = C_\alpha(1 - \psi) + C_i^k(r, z, t = t_{sa})\psi, \tag{9}$$

$$C(r, z = 0, t) = C_i^k(r, z = 0, t);\ \partial C/\partial z|_{z = \text{ZLI}} = 0, \tag{10}$$

where $C^k{}_i$ is the carbon source from decaying carbides, which decay in accordance with the equilibrium phase diagram for carbon steel carbides. This admits the analytical interpretation [13]:

$$C_i^k(T(r, z, t)) = 0{,}8 + 0{,}002\,(T(r, z, t) - 723^\circ C). \tag{11}$$

In (8) and (9), ψ is the random probability function of the carbide particle distribution at the point (r, z) in the ZLI. It takes on the value $\psi = 1$ if there is carbide at the point (r, z); otherwise $\psi = 0$. Applying consecutively the integral Laplace and Hankel transforms to (8), we obtain

$$\begin{aligned} C_{\delta,s}(\delta, z, s) &= (0{,}002T(\delta, z = 0, s) - 0{,}646 r j_1(\delta r))\exp(-z\sqrt{\delta^2 + s/D}) - \\ &- (C_\alpha(1 - \psi) + 0{,}646\psi)\frac{r j_1(\delta r)}{D\delta(\delta^2 + s/D)}[\exp(-z\sqrt{\delta^2 + s/D}) - \\ &- 1] - \frac{A I_0 r_\omega^2 a}{2\lambda_s\sqrt{\delta^2 + s/a}}\, 0{,}001/s/(a - D) * \\ &* \exp[-\delta^2 r_n^2/4 - z(\sqrt{\delta^2 + s/a} + \sqrt{\delta^2 + s/D})], \end{aligned} \tag{12}$$

where $T_{\delta,s}$ is from the solution to the heat problem. The explicit functional dependence of the carbon concentration on the ZLI coordinates and the laser quenching parameters can now be found by using the Laplace and Hankel transforms. We note that the expression obtained is quite cumbersome. It makes sense to simplify it by using the physically reasonable approximation $D \ll a$. Then

$$\begin{aligned} C(r, z, t) &= 0{,}002T(r, z, t) + C_\alpha(1 - \psi) - 0{,}646\psi - \\ &- 0{,}323\int_0^\infty r j_1(\delta r) j_0(\delta r)\, d\delta * \left[\exp(-z\delta)\operatorname{erfc}\left[\frac{z - 2\delta D(t - t_{sa})}{2\sqrt{D(t - t_{sa})}}\right] + \right. \\ &\left. + \exp(z\delta)\operatorname{erfc}\left[\frac{z + 2\delta D(t - t_{sa})}{2\sqrt{D(t - t_{sa})}}\right]\right], \end{aligned} \tag{13}$$

It is easy to show that (13) satisfies the initial and boundary conditions of the diffusion equation (8)-(10).

The elastic thermal stresses which arise during laser heating disappear with subsequent cooling if these stresses do not exceed σ_f. It is assumed that stress and strain can exist in the initial state of the metal, before LA, but heating up to laser temperatures completely removes these initial stresses [11, 14]. Besides, in problems of laser quench-hardening of steels it can be assumed that the rate of establishment of the stress state is higher than the rate of establishing thermal equilibrium, that is, $\sqrt{a\tau_d} \ll v_s\tau_d$ [2]. Then for long duration LA τ_d characterized by pulsed and continuous radiation, $d^2u_i/dt^2 = 0$ and the equation of mechanical motion from elasticity theory [15] is stationary. Unlike the traditional approach to analysis of the thermoelastic state of steel during LA, which considers only elastic and thermal stresses, we will take other components of the total strain tensor $\varepsilon_{ij}(x_c, t)$ into our calculation. So it is valid to suppose that at the moment of phase transformation ($\alpha \to \gamma$) there are phase stresses due to the difference in specific volumes of the α- and γ-phases of Fe. The subsequent course of the phase transition (e.g., $\alpha \to \gamma$ during laser heating) leads to carbon saturation of the γ-phase Fe lattice. As a result, the lattice parameter increases [12]. The growth of the lattice parameter within the framework of the existing new phase leads to yet greater increase of its relative volume compared to the surrounding old phase (i.e., α-phase), and in analogy with thermal stresses, it makes sense to speak of concentration stresses [12]:

$$\sigma_{ik}^{\text{con}}(x_c, t) = -3K\beta(C(x_c, t) - C_0), \tag{14}$$

where β is the relative change in the lattice parameter of the solvent in a solution of 1% by mass of additive, and $C(x_c, t)$ is the concentration of these additives. In this way, elasticity theory allows for a complex description of elastic, thermal, concentration and phase stresses:

$$\sigma_{ik}(x_c, t) = -3K\beta(C(T(x_c, t)) - C_0)\delta_{ik} - 3K\varkappa(T(x_c, t) - T_0)\delta_{ik} + + K\varepsilon_{ll}(x_c, t)\delta_{ik} + 2\mu(\varepsilon_{ik}(x_c, t) - 1/3\varepsilon_{ll}(x_c, t)). \tag{15}$$

Taking these assumptions into account, the elastic problem of determining the stresses and strains reduces to solution of the following system of equations:

$$\begin{gathered}\partial\sigma_{rr}/\partial r + \partial\sigma_{rz}/\partial z + (\sigma_{rr} - \sigma_{\theta\theta})/r = 0,\\ \partial\sigma_{rz}/\partial r + \partial\sigma_{zz}/\partial z + \sigma_{rz}/r = 0,\end{gathered} \tag{16}$$

for which the stresses and strains are expressed in terms of displacements as:

$$\begin{gathered}\sigma_{rr} = (\lambda + 2\mu)\,\partial u_r/\partial r + \lambda(u_r/r + \partial u_z/\partial z) - (3\lambda + 2\mu)(\varkappa T + \beta C),\\ \sigma_{\theta\theta} = \lambda(\partial u_r/\partial r + \partial u_z/\partial z) + (\lambda + 2\mu)\,u_r/r - (3\lambda + 2\mu)(\varkappa T + \beta C),\\ \sigma_{zz} = (\partial u_r/\partial r + u_r/r)\,\lambda + (\lambda + 2\mu)\,\partial u_z/\partial z - (3\lambda + 2\mu)(\varkappa T + \beta C),\end{gathered} \tag{17}$$

$\sigma_{rz} = 2\mu\varepsilon_{rz}$, $\varepsilon_{rr} = \partial u_r/\partial r$, $\varepsilon_{\theta\theta} = u_r/r$, $\varepsilon_{zz} = \partial u_z/dz$, $\varepsilon_{rz} = (\partial u_z/\partial r + \partial u_r/\partial z)/2$, and all remaining components of the stress and strain tensors are evidently zero. The condition at the free boundary $z = 0$ is as usual $\sigma_{zz} = \sigma_{rz} = 0$. The system (16) and (17) was solved by the method of thermoelastic potentials with the aid of Hankel transforms and using the solution for the thermal and diffusion problem found earlier: (4), (6), and (12). Finally, for the displacement and strains we obtain:

$$\begin{aligned}
u_r &= \int_0^\infty j_1 d\delta\,[(1 + \nu)\exp(-\delta z)(\varkappa T_\delta + \beta C_\delta)[1 - z\delta/(1 - 2\nu)] + \delta A\,[\operatorname{ch}\delta z -\\
&\quad - z\delta\exp(-\delta z) + (1 - 2\nu)\exp(-\delta z)] - \delta B];\\
u_z &= -\int_0^\infty j_0 d\delta\,[(1 + \nu)\exp(-\delta z)(\varkappa T_\delta + \beta C_\delta)[1 + (1 + z\delta)/(1 - 2\nu)] +\\
&\quad + \delta A(\operatorname{ch}\delta z + z\delta\exp(-\delta z) + (1 - 2\nu)\exp(-\delta z)] - \partial B/\partial z];\\
\varepsilon_{rr} &= \int_0^\infty (\delta j_0 - j_1/r)\,d\delta\,[(1 + \nu)\exp(-\delta z)(\varkappa T_\delta + \beta C_\delta)[1 - z\delta/(1 - 2\nu)] +\\
&\quad + \delta A * [\operatorname{ch}\delta z - z\delta\exp(-\delta z) + (1 - 2\nu)\exp(-\delta z)] - \delta B];\\
\varepsilon_{zz} &= \int_0^\infty j_0\delta d\delta\,[(1 + \nu)\exp(-\delta z)(\varkappa T_\delta + \beta C_\delta)[1 + z\delta/(1 - 2\nu)] + \delta A\,\times\\
&\quad \times [z\delta\exp(-\delta z) - \operatorname{sh}\delta z + (1 - 2\nu)\exp(-\delta z)] - \exp(-\delta z)] + 1/\delta\,\partial^2 B/\partial z^2];\\
\varepsilon_{rz} &= 2\int_0^\infty j_1\delta d\delta\,[(1 + \nu)\exp(-\delta z)(\varkappa T_\delta + \beta C_\delta)\,z\delta/(1 - 2\nu) + \delta A\,[\operatorname{sh}\delta z + z\delta\, *\\
&\quad * \exp(-\delta z)] + \partial B/\partial z];\quad \varepsilon_{\theta\theta} = u_r/r.
\end{aligned} \tag{18}$$

The explicit form of functions $A(\delta)$ and $B(\delta, z)$ is determined in the Appendix. Solution (18) completely satisfies the boundary conditions at the free surface and determines the stress tensor components for nonstationary LA. From inspection it also follows that $\varepsilon_{rr} + \varepsilon_{zz} + \varepsilon_{\theta\theta} = 3(\varkappa T + \beta C)$. Due to the cylindrical symmetry of the problem, u_r, $\varepsilon_{\theta\theta}$, ε_{rz}, and σ_{rz} vanish at $r = 0$.

However, (18) cannot be used when the plastic state is reached, when the stresses are $\geq\sigma_f$. Analysis of the stresses and strains in the plastic region of ZLI is in principle necessary, since only the existence of plastic strain explains the presence of residual stresses during cooling. The best-known condition for the change in a metal to a state of flow is the von Mises criterion [11, 14]

$$(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{rr})^2 = 2\sigma_f^2. \tag{19}$$

Below we will rely on one of the versions of the elastic—plastic approximation method of Il'yushin for the solution of the nonlinear "deformation" equations of the theory of plastic equilibrium. Then the problem of plasticity reduces to a solution of the previously given sequences of linear equations of the type (16) and (17) with an accuracy $\eta \geq (\gamma^j{}_i - \gamma^{j+1}{}_i)/\gamma^j{}_i$. Each of these can be interpreted as a Hookean elastic problem [14, 16]. The distinguishing feature of the method used here of variable elastic parameters is that it allows us to find a solution in the plastic region of ZLI without changing problem statement (16) and (17), but with variable elastic coefficients [16]:

$$\begin{gathered} E^* = 3E/(2(\nu+1)\Phi + 1 - 2\nu), \\ \nu^* = \frac{\Phi(1+\nu) - 1 + 2\nu}{2\Phi(1+\nu) + 1 - 2\nu}, \\ G^* = E^*/2(1+\nu^*) = G/\Phi. \end{gathered} \tag{20}$$

We note that for $\nu \to 1/2$ (a rigid-plastic body), the relations (20) simplify: $\nu^* = 1/2$; $E^* = E/\Phi$. The explicit form of the universal plasticity function Φ, determined from a series of experiments on proportional (simple) loading for constant γ_i and T over the course of an experiment, was taken from [17] using the temperature dependence for σ_f:

$$\begin{gathered} \sigma_f = 5q\exp(-0{,}005(T - 1200))\ (\text{MPa}), \\ \Phi(T, \varepsilon, \gamma_i) = \sigma_f[1 + (3 + 6\varepsilon^{0,25189})\gamma_i^{0,29} * \exp(0{,}016(T - 1200))]/G, \\ \gamma_i = [[(\varepsilon_{rr} - \varepsilon_{\theta\theta})^2 + (\varepsilon_{\theta\theta} - \varepsilon_{zz})^2 + (\varepsilon_{zz} - \varepsilon_{rr})^2 + \varepsilon_{rz}^2/3]/3]^{1/2}, \end{gathered} \tag{21}$$

where q = 1 for steel 45, q = 0.85 for U8 and q = 1.1 for steel 40.

A package of FORTRAN programs based on this theoretical model has been written. It includes: computation of the thermal field in space during the heating and cooling stages; determination of the boundaries of ZLI and zone of laser tempering (ZLT) with depth and at the sample surface using (5) and (7); space—time distributions of free carbon in the γ-phase in ZLI according to (13); and computations using (17) and (18) of the displacements, strains and stresses in the elastic regions of ZLI and ZLT. Finally, an iterative mesh scheme for computing displacements, strains and stresses in the plastic ZLI regions was developed and implemented in the package. This iterative scheme uses the method of variable elastic parameters (20) within the framework of "deformation" theory. This permits determination of the linear dislocation density and evaluation of the amount of residual stress.

We have done calculations for the carbon steels steel 45 and U8 in specific LA regimes [5-7] without heat polishing of the surface by the pulsed and continuous radiation. Figure 1a shows an example of the thermal calculation at the ZLI boundary (dashed lines) and the ZLT (solid line). It also shows the position of these boundaries with respect to the mesh partitioning in r and z. The points indicate those mesh nodes where, according to the probability function ψ, carbide is to be found. The hatched area is the zone of plastic deformation during heating, and the cross-hatched area shows the zone of secondary plastic deformation during cooling. Figure 1b depicts the distribution of carbon concentration in the γ-phase ZLI. This distribution is characterized by a "trough" in the functional dependence C = C(r, z) at those mesh nodes with the highest probability of carbide particle location. This is completely justified if the carbon at these points must be in a bound state. The calculations show that the u_z vector displacement component is always negative. In this case it has maximum absolute value at the center of the spot equal to $u_z = -8.85$ μm at the moment of LA cessation ($T_{max} = 1380$°C).

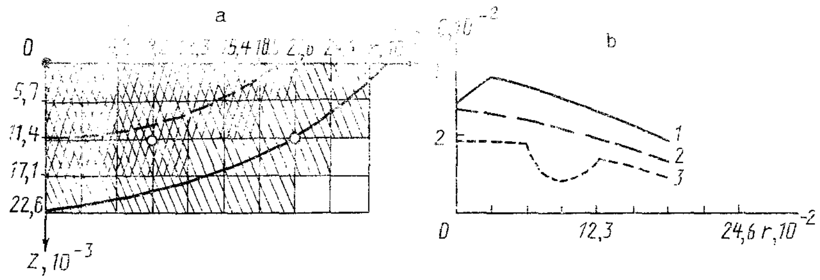

Fig. 1. Computational results for a (a) thermal (°C) and (b) diffusion (% by mass) problem (regime [5]): 1) $z = 0$ cm; 2) $z = 5.7 \cdot 10^{-3}$ cm; 3) $z = 11.4 \cdot 10^{3}$ cm. r, z in cm, C in % by mass.

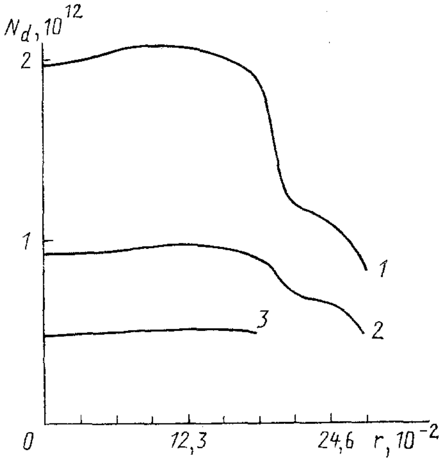

Fig. 2

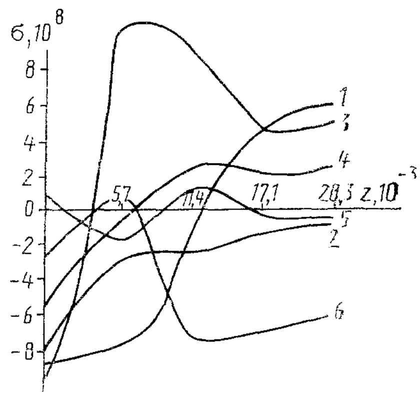

Fig. 3

Fig. 2. Computation of the lineal dislocation density ($1/cm^2$) (regime [7]): 1) $z = 0$ cm; 2) $z = 5.95 \cdot 10^{-2}$ cm; 3) $11.9 \cdot 10^{-2}$ cm.

Fig. 3. Comparison of calculated stress (N/m^2) with experiment (regime [5]): 1) experiment; 2) diagonalization; 3) plus averaging; 4) plus accounting for unloading; 5) $\beta = 0$; 6) original calculation.

Physically this is expressed in the mechanical displacement of the free surface. The calculation already gives $u_z(r = 0, z = 0, t) = -0.09$ μm at room temperature during the cooling stage, which once more gives evidence of the insignificant amount of distortion (same LA regime as in [5]). Knowledge of the elastic–plastic strains permits calculation of N_d in ZLI (Fig. 2). To do this it is necessary to subtract out the elastic contribution from the elastic–plastic strains during maximum heat flux. Then $N_d = \varepsilon_{pl}/(b \cdot \ell)$ [1], where $b \simeq 4 \cdot 10^{-8}$ cm. The quantity ℓ is of the same order as the grain size, and if we assume that we are dealing with Frank–Read sources, then such a characteristic size is intrinsic to the model, since $\sigma_f/G = b/\ell$. The calculation using these relations gives the qualitatively correct results of Fig. 2, since σ_f (21) and G are computed at every ZLI point. However, comparison of our computational results with available experimental data on residual stress distribution with depth Z and at the surface R of the ZLI [5-7] shows significant disagreement (e.g., see Fig. 3, curves 1 and 6 for the regime being discussed [5]).

A $\sin^2 \psi$ analysis of the experimental procedure for measuring residual stresses, whose contents are given in detail in [18], shows the following. For adequate comparison of computational results with experiment, we must: diagonalize the matrix components of the strain tensor, changing the direction of the z-axis in the process; carry out an area averaging on the x-ray photographs; and change over to a plane-stress state, that is, account for unloading. Figure 3 (curves 2-4) show the results of carrying out these operations in succession. As can be seen, qualitative agreement of computational results with experiment has been practically achieved. In principle, the important result is depicted in Fig. 3 (curve 5), where, other conditions being equal, the residual stresses in a purely thermoelastic–plastic problem were computed, without accounting for carbon diffusion ($\beta = 0$). Comparison of curves (1, 4, 5) shows the significance of introducing the concentration stresses in our work.

Thus in this work, we have done the following.

1) We have obtained a complete analytical solution to the uncoupled quasistationary thermoelastic problem which includes concentration stresses.

2) We developed an iterative mesh scheme for computing the values of the lineal dislocation density and residual stresses. This was done using the method of variable elastic parameters.

3) We obtained full agreement of the thermal calculations with experimentally determined boundaries of ZLI and ZLT and qualitative agreement for the residual stress calculation.

APPENDIX

Equations (16) and (17) are easily reduced to a system of second order differential equations by using the Gooder function method [11]. It can be shown that:

$$A(\delta) = \Omega \int_0^\infty S_\delta \exp(-\delta\xi)\, d\xi,$$
$$B(\delta, z) = \Omega \int_0^\infty S_\delta \operatorname{sh} \delta(z-\xi)\, d\xi, \tag{1A}$$

where $S_\delta = \varkappa T_\delta + \beta C_\delta$. Then, using (5) and (12) we have during the heating stage:

$$\Omega = \frac{(1+\nu)}{(1-\nu)} (\varkappa + 0{,}002\beta) \frac{AI_0 r_s^2}{4\lambda_s \delta} \exp(-\delta^2 r_s^2/4);$$
$$A(\delta) = \Omega[1/\delta - 2\sqrt{at/\pi} \exp(-\delta^2 at) - \delta at * \operatorname{erfc}(\delta\sqrt{at})]; \tag{2A}$$
$$B(\delta, z) = \Omega[\operatorname{sh}\delta z * (1/\delta + 2\delta at) + 2\sqrt{at/\pi} \exp(-z^2/4at - \delta^2 at) - 2\operatorname{ch}\delta z *$$
$$* \sqrt{at/\pi} \exp(-\delta^2 at) + \delta at \operatorname{erf}(\delta\sqrt{at}) - \exp(-\delta z)(z - 2\delta at)/2 \operatorname{erfc}[(z -$$
$$- 2\delta at)/2\sqrt{at}] - \exp(\delta z)(z + 2\delta at)/2 \operatorname{erfc}[(z + 2\delta at)/2\sqrt{at}]],$$

and for the cooling stage, using (7) and (13):

$$\Omega = \frac{AI_0 r_s^2}{8\lambda_s \delta} \exp(-\delta^2 r_s^2/4) \frac{(1+\nu)}{(1-\nu)} (\varkappa + 0{,}002\beta) \int_0^\infty d\xi [\exp(-$$
$$- \delta\xi) \operatorname{erfc}[(\xi - 2\delta a\tau_d)/2\sqrt{a\tau_d}] - \exp(\delta\xi) \operatorname{erfc}[(\xi + 2\delta a\tau_d)/2\sqrt{a\tau_d}]];$$
$$A(\delta) = \Omega\left[\exp(-\xi\delta) \operatorname{erfc}\left[\frac{2\delta a(t-\tau_d) - \xi}{2\sqrt{a(t-\tau_d)}}\right] + \exp(\xi\delta) \operatorname{erfc}\left[\frac{2\delta a(t-\tau_d) + \xi}{2\sqrt{a(t-\tau_d)}}\right]\right]; \tag{3A}$$
$$B(\delta, z) = 2\Omega \left| 2\operatorname{sh}\delta z * \operatorname{ch}\delta\xi - \operatorname{ch}\delta z \left[\exp(-\xi\delta) \operatorname{erfc}\left[\frac{2\delta a(t-\tau_d) - \xi}{2\sqrt{a(t-\tau_d)}}\right] + \right.\right.$$
$$\left.\left. + \exp(\delta\xi) \operatorname{erfc}\left[\frac{2\delta a(t-\tau_d) + \xi}{2\sqrt{a(t-\tau_d)}}\right]\right]\right|,$$

where ξ is the integration variable in (7) and (3A). Further, by simple differentiation of the third equations in (2A) and (3A) we can easily find $\partial B/\partial z$ and $\partial^2 B/\partial z^2$. We note that $A(\delta)$ and $B(\delta, z)$ do not depend on the radius r, and in addition, $B(\delta, z = 0) = \partial B(\delta, z = 0)/\partial z = \partial^2 B(\delta, z = 0)/\partial z^2 = 0$, which is evident from the second equation in (1A).

NOTATION

T_j, representative temperature in the steel; T_{mp}, melting point temperature for the steel; I_0, maximum power density of the LA; A, LR absorption coefficient; a, thermal diffusivity; λ_s, thermal conductivity; ρ, density; r_s, radius of the LR heating spot; τ_d, duration of the LA; E, Young's modulus; ν, Poisson's ratio; $\varkappa$, thermal expansion coefficient; $j_0(x)$, $j_1(x)$, Bessel functions of the zeroth and first order; T_{sa}, temperature of the start of austenizing of the steel; C, concentration of free carbon in the γ-phase ZLI; R_0, Z_0, ZLI boundary coordinates; D, diffusion coefficient for carbon in the γ-phase; T_{sa}, start time for the $(\alpha \to \gamma)$ transformation at the ZLI boundary; C_α, concentration of carbon in ferrite ~ 0.02% by mass; v_s, speed of sound in the metal; σ_f, flow limit for steel; ε_{ij}, strain tensor; σ_{ij}, stress tensor; K, coefficient of bulk compression; λ, μ, Lamé coefficients; δ_{ij}, Kronecker delta; $G = \mu$, shear modulus; γ_i, strain rate; Φ, universal plasticity function; N_d, lineal dislocation density; b, Burgers vector; ℓ, dislocation path.

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